Mathematics in Ancient Greece

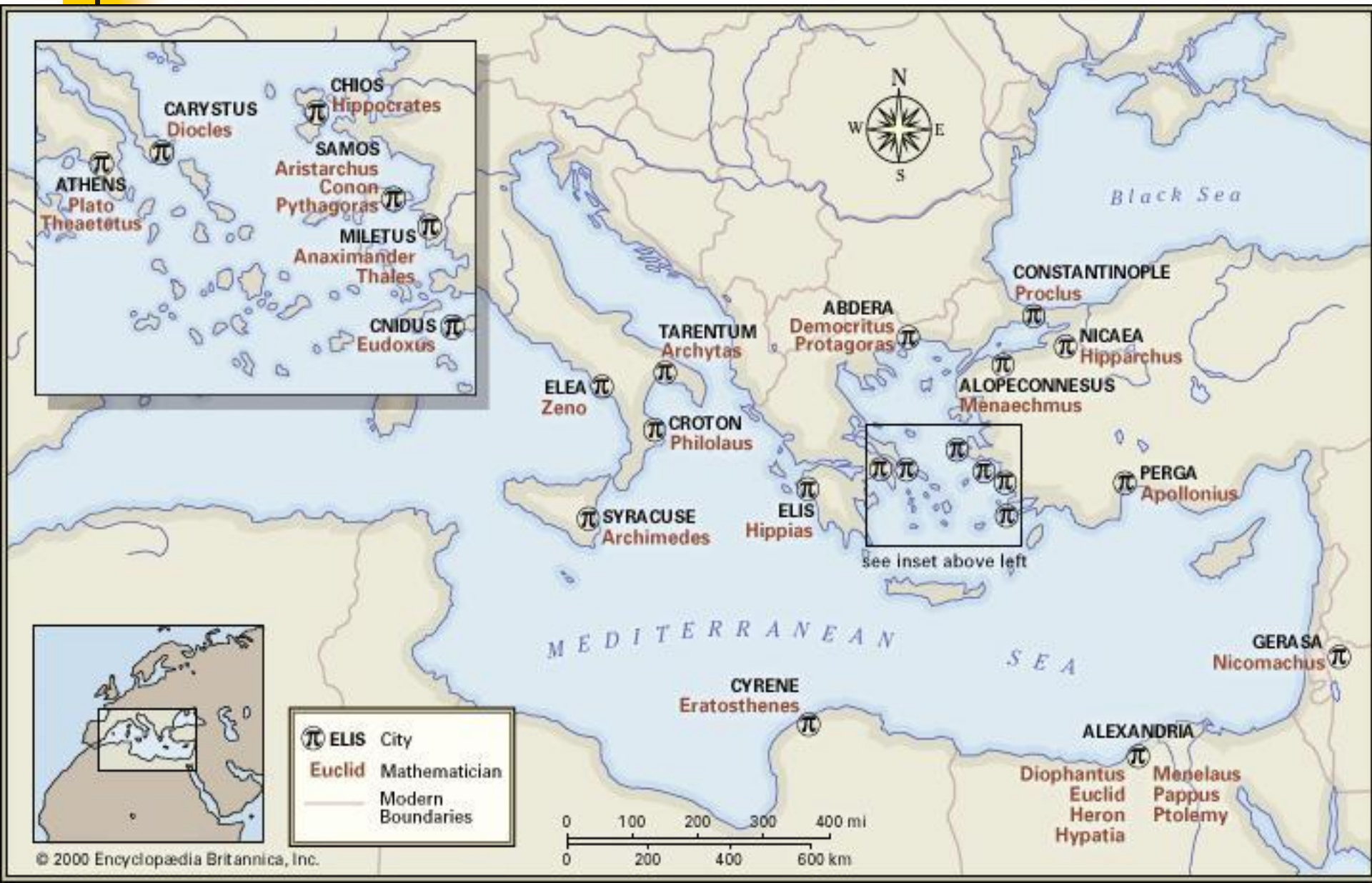
Let none ignorant of Geometry enter here

Plato's academy

picture by Michelangelo

All is number

Maths in Ancient Greece

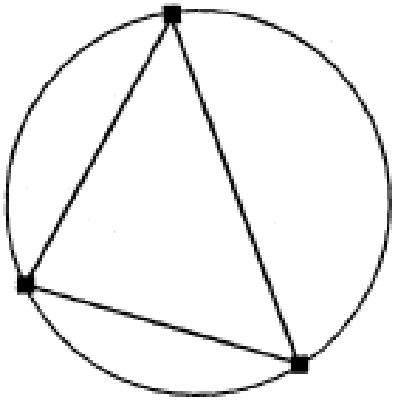


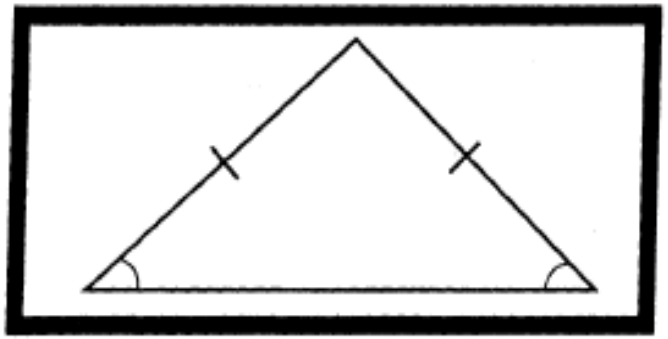
Thales - Θαλής

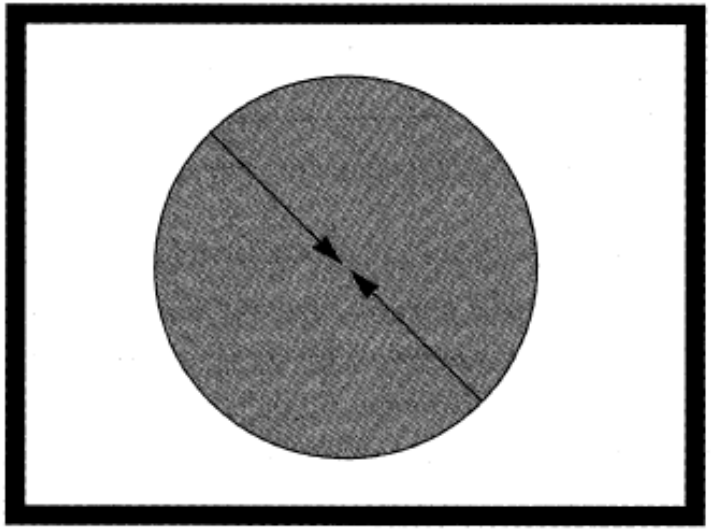
Thales was named (afterwards) "***father of geometry***"
He was born in Militos (a city in Minor Asia across the island of Samos) at around 620 B.C.

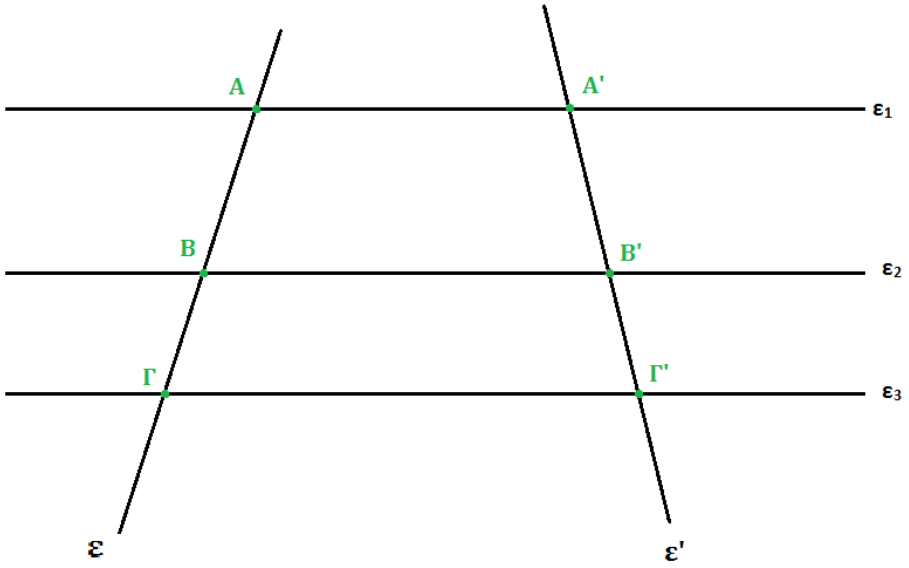
He was one of the Seven ***"wise men"*** of the ancient times and famous by the saying "***Know yourself***" and he formed the first rules about mathematical objects

Thales - Θαλής









Thales - Θαλής

He was forever asking "**Why**?" and working out his answer from what he saw, and standing ready to prove it.

He was a **great observer**
Being also a *merchant* he travelled a lot around the known world from Asia Minor to Mesopotamia (Babylon), and from Magna Grecia to Egypt

HOW HIGH IS THE PYRAMID?

In the Land of the river Nile, Thales amazed and frightened his guides by telling them, as if by magic, the exact height of the Great Pyramid

The story is worth reviewing in some detail.
Naturally, Thales' visit to **Egypt** was not complete without a sightseeing trip to the desert at Giza, to see the three pyramids and the Sphinx half-buried in the sand nearby. In 600 **B.C.** the pyramids were about **2000** years old

Thales - Θαλής

Thales stood for a time admiring the most gigantic of the tombs: the **Great Pyramid of Cheops**,

Then he asked his celebrated question.

"***How high is this pyramid***?"

Thales - Θαλής

While the argument went on, Thales and his friend had been walking around quietly, staying close to the pyramid's shadow, where it was cool. "Never mind my question!" called Thales, as the guides approached.

"I know the answer. The Great Pyramid at Giza rises to a height of **276 paces**!“

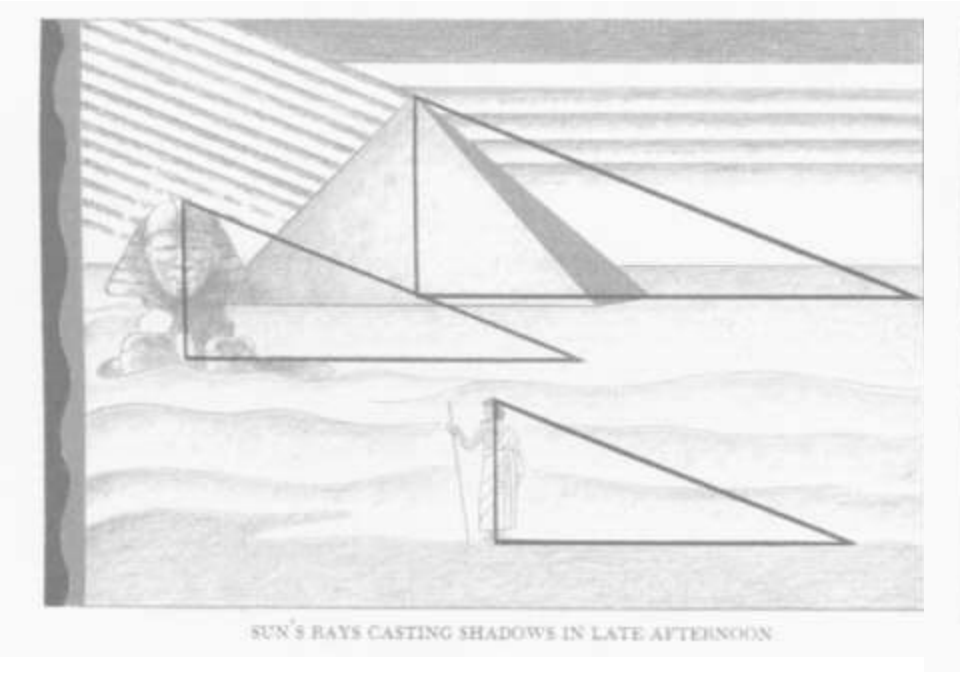
πήχης łokieć długość łokcia (46,2 or 52 cm)

Smaller units of length

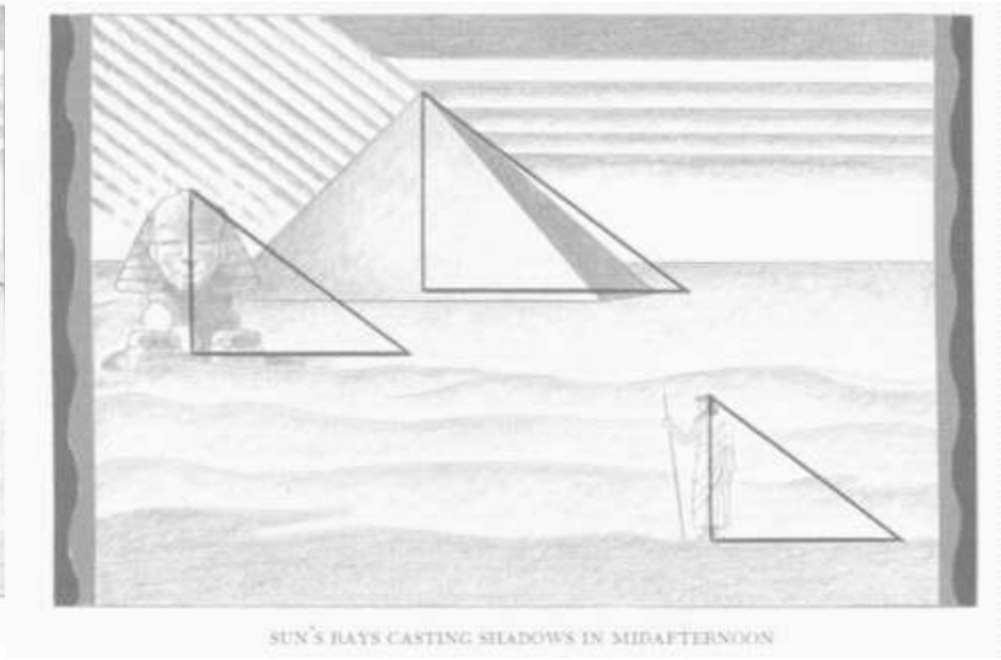
Unit	Greek name	Equal to	Metric equivalent	Description
pēchys	πῆχυς	24 daktyloi	462.3 mm (18.20 in)	cubit

Thales - Θαλής

Thales did not get the answer by magic.
He simply measured two shadows on the sand, and then used an ***abstract rule*** from his new kind of geometry

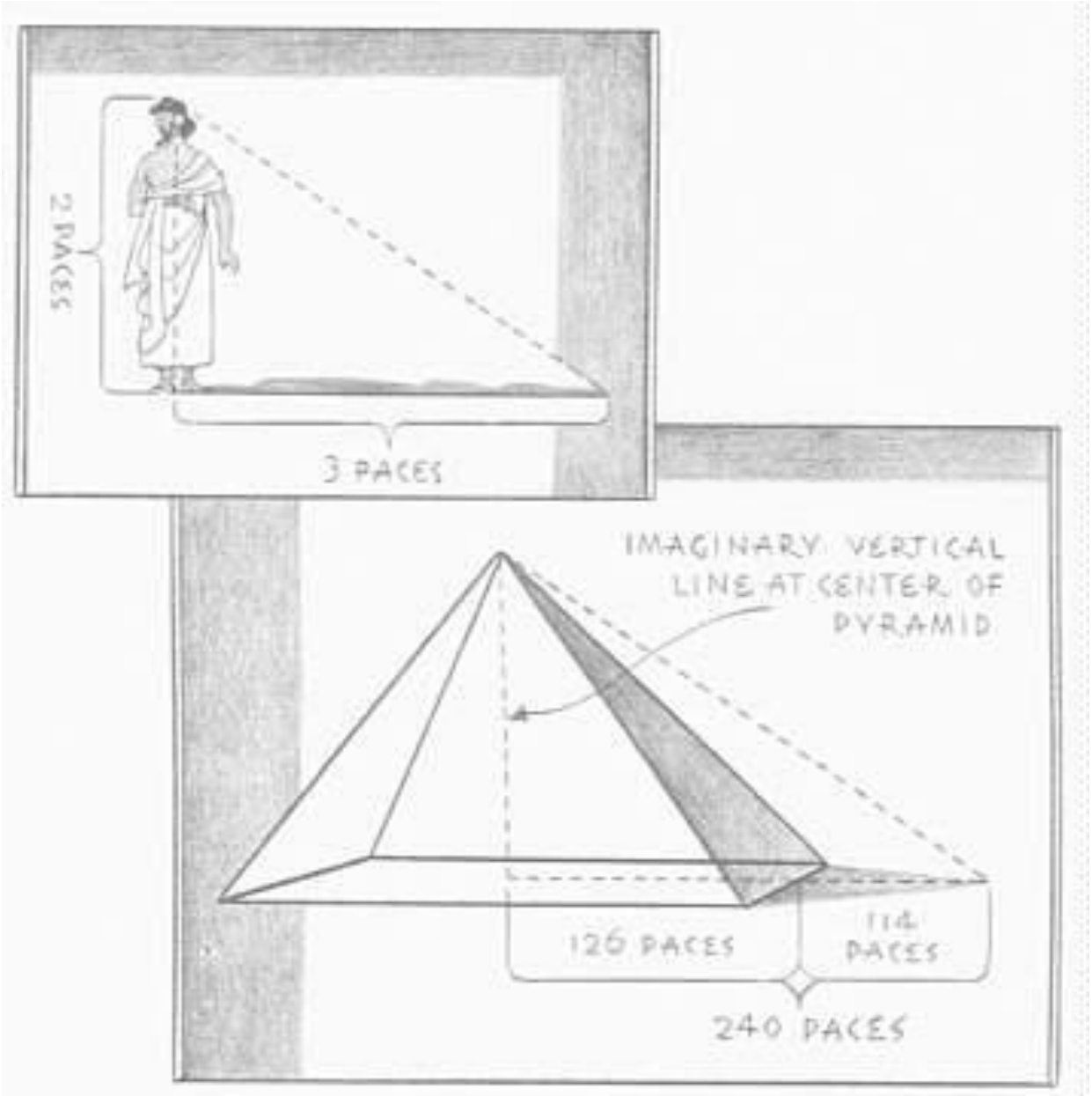


SUN'S RAYS CASTING SHADOWS IN LATE AFTERNOON

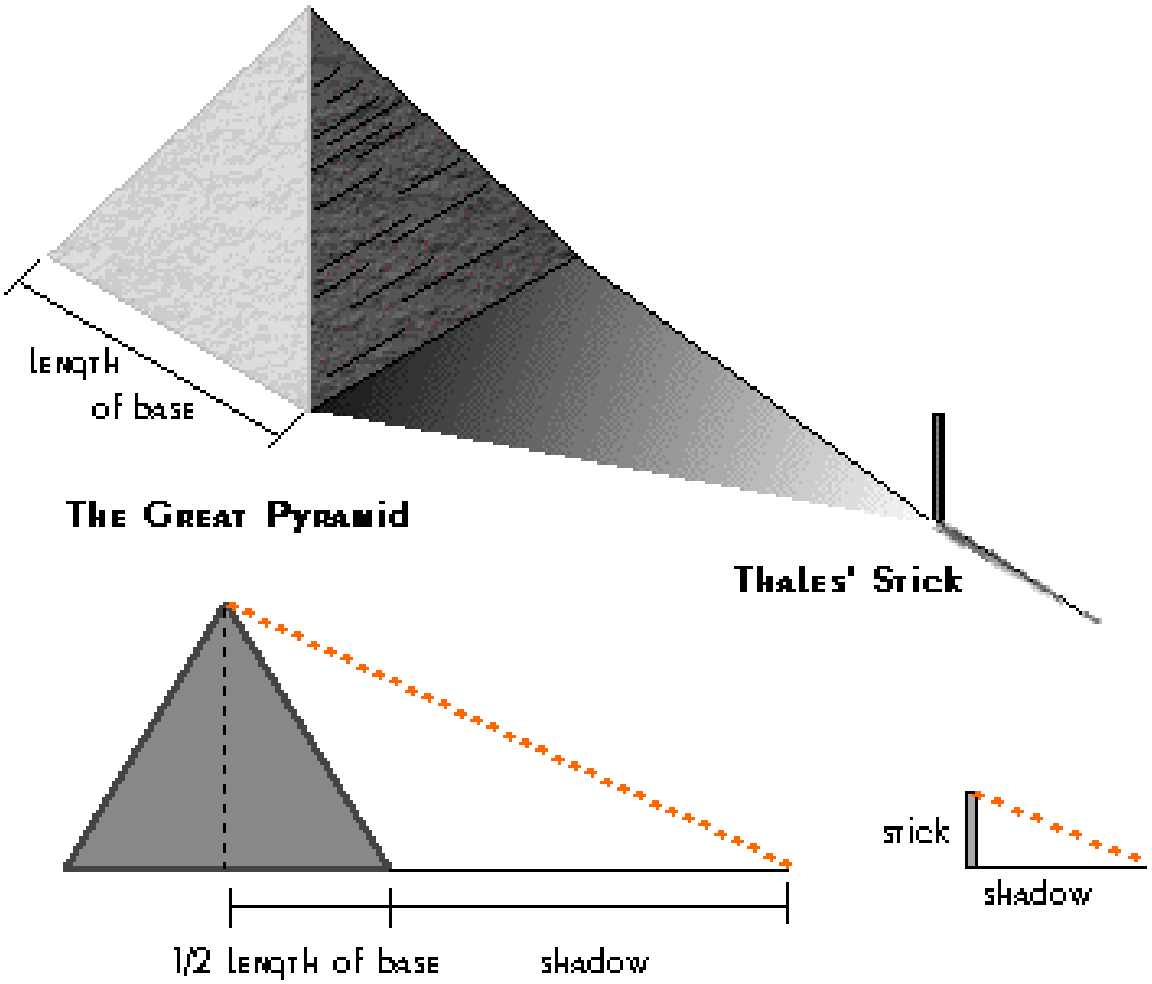


SUN'S RAYS CASTING SHADOWS IN MIDAFTERNOON

Thales - Θαλής

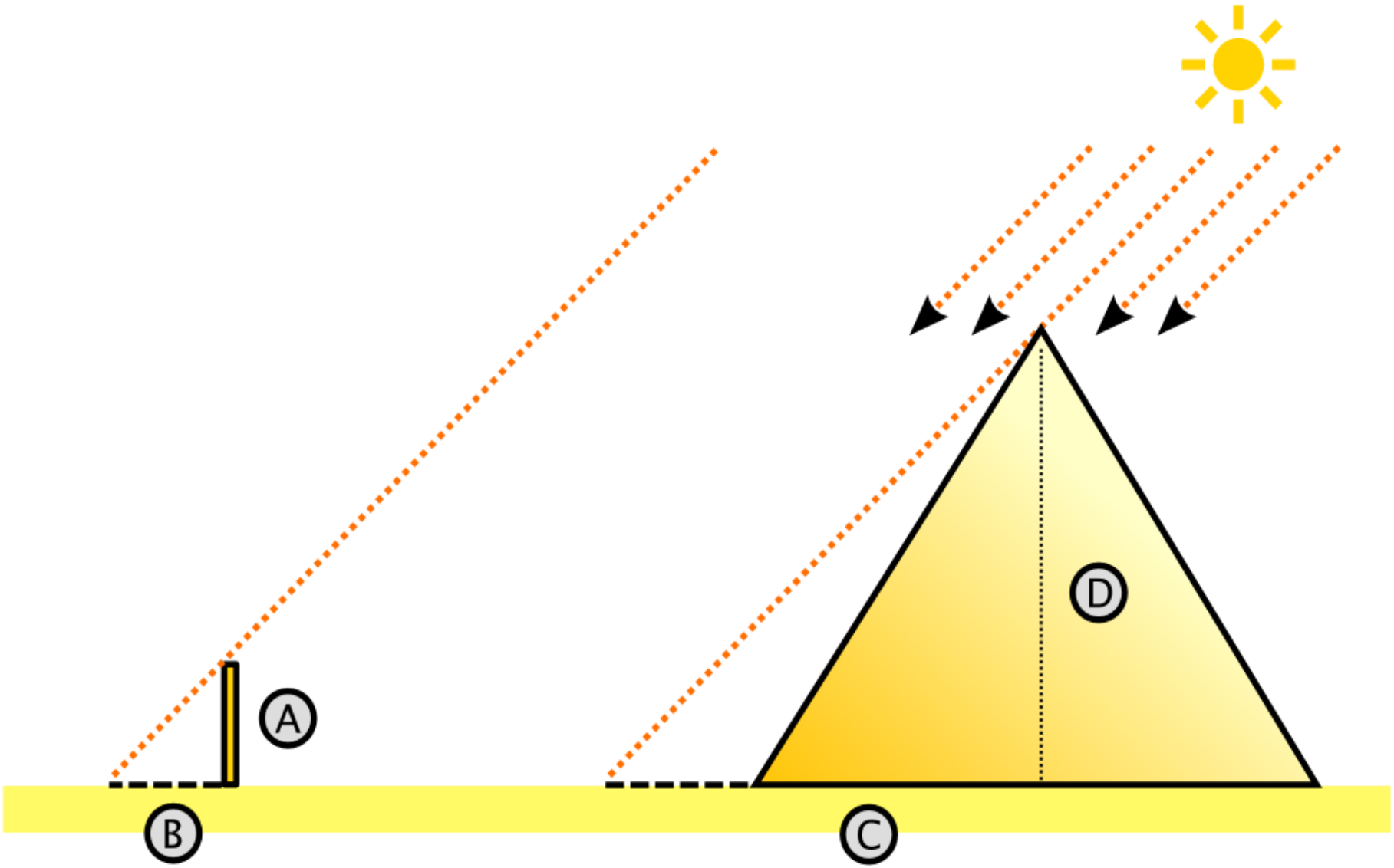


Thales - Θαλής



$$\frac{\text{height of pyramid}}{\text{1/2 length of base + shadow of pyramid}} = \frac{\text{height of stick}}{\text{shadow of stick}}$$

Thales - Θαλής



Thales - Θαλής

The Pyramid of Cheops is in Giza in Latitude 30^0 N as we are, but closer to equator than us

Astronomers have calculated that *twice a year* in that latitude when the sun rays have 45^0 degrees angle the shadow of an object is the same to the height of that object
Thales used a robe and he needed a measure unit (some say he used himself as a unit others say he used a *pace*)

He found the height of the pyramid to be 85 Thales or **276,25** *paces*

Today we know that the real height is **280** *paces* or 147 meters

Another big accomplishment is that he predicted a **solar eclipse**

Pythagoras – Πυθαγόρας all is number

Pythagoras was born in **Samos** a Greek island near the shores of today's Turkey in 6th century b.C.

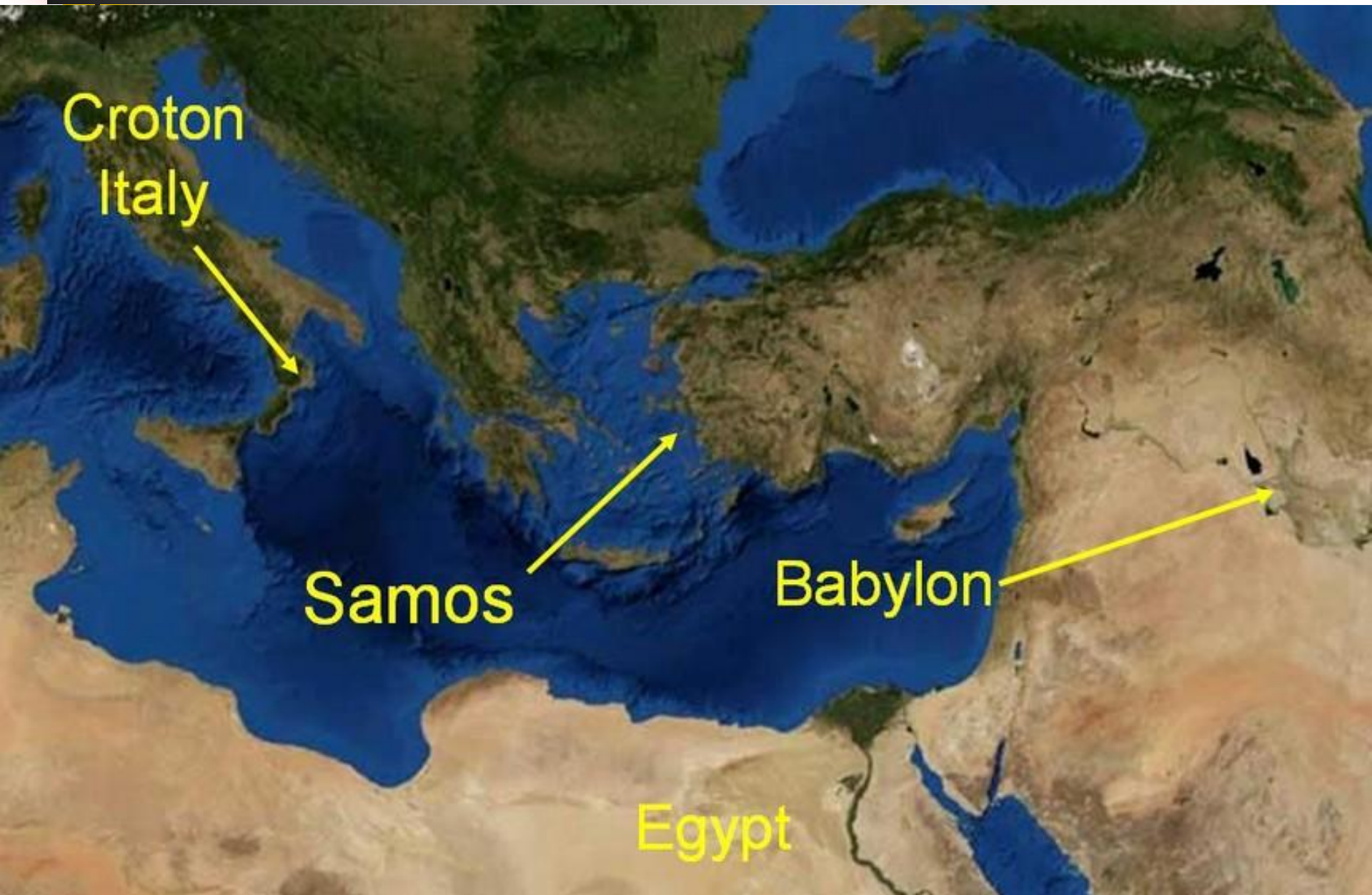
He is known as

"the man who was seeing numbers everywhere"

After almost 40 years he returns to Samos for a while and finally emigrates to southern Italy (magna Grecia) where he founded his "school" in Croton.

A famous secret society that contributed a great deal to the development of geometry. We might call it the world's **first mathematics club**

Pythagoras – Πυθαγόρας



Pythagoras - Πυθαγόρας

He was well educated; he was a **fine musician**, he played the lyre and used music to help people who were ill; he **learned poetry** and was able to recite famous and popular Greek writers like Homer.

Although he was a very important figure in the development of mathematical ideas, we don't know much about Pythagoras' actual mathematical achievements.

Pythagoras was the first to distinguish between **odd** and **even** numbers and he came up with the idea of **prime numbers** and showed how they were different from what we call **composite numbers**.

Pythagoras - Πυθαγόρας

Odd numbers, like 1, 3, 5, 7, 9, 11

parzystych i nieparzystych

Even numbers, like 2, 4, 6, 8, 10, 12

Triangular numbers, like 1, 3, 6, 10, 15

Square numbers, like 1, 4, 9, 16, 25

But ***ten*** was the very best number of all: it was the sum of the first four integers one, two, three, and four [1 + 2 + 3 + 4 = 10].
If each of these were represented by a dot notation in a series, one above the other, they formed a perfect triangle.

Pythagoras - Πυθαγόρας

Some of the things they believed about numbers seem odd to us now. For example, numbers were thought to have their own special nature. Pythagoreans thought numbers were male or female, ugly or beautiful, or had a special meaning.

1 is the number of **reason**.

2 is the first even or **female number**, the number of opinion.

3 is the first true **male number**, the number of harmony.

4 is the number of **justice** or retribution.

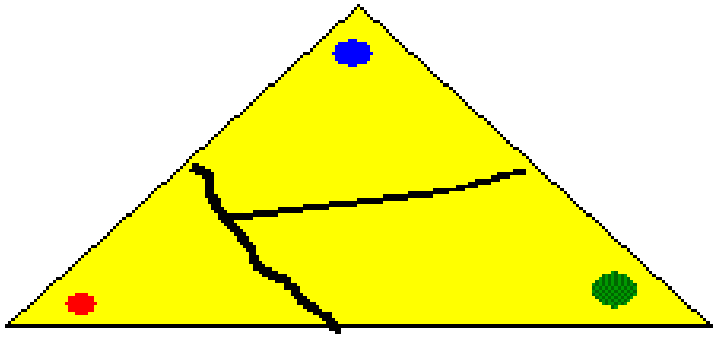
5 means **marriage**.

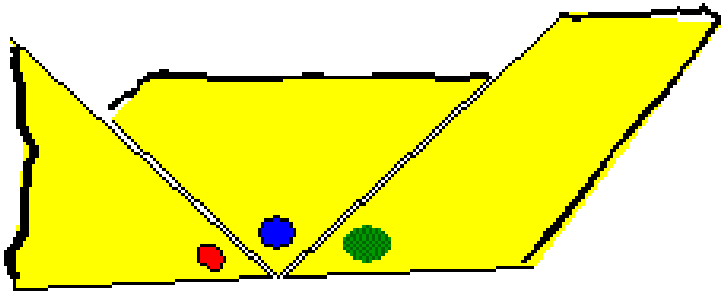
6 is **creation** ...and so on.

Pythagoras also studied shapes and were particularly interested in triangles.
One theorem (a statement of an idea) they worked on was:

The sum of the angles of a triangle is equal to two right angles .

This means that if you take any triangle, tear off the corners and fit them together like a puzzle, you will make a straight line (that is the same as two right-angles).





Pythagorean Theorem

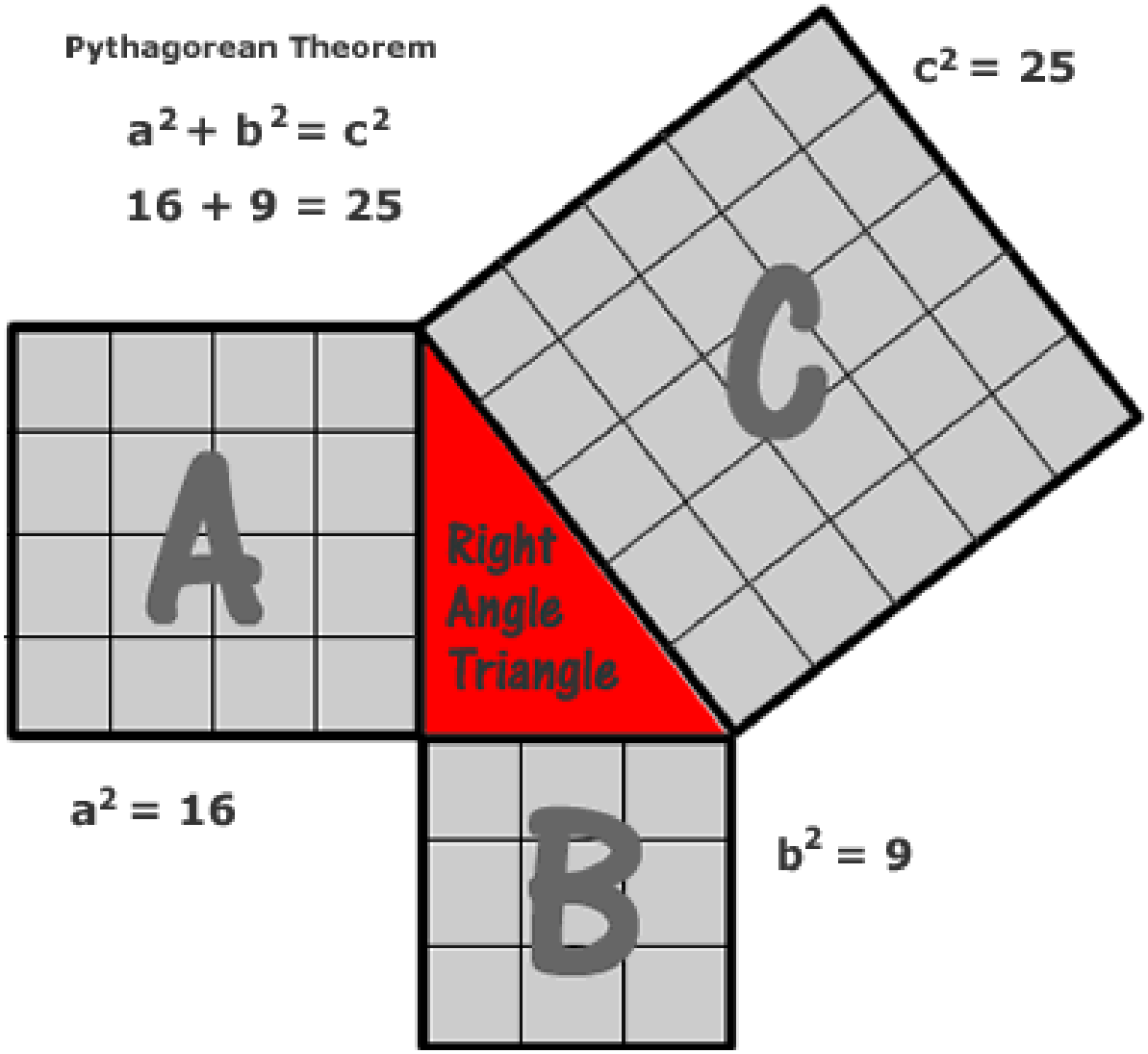
The Pythagorean theorem says:

*In any **right triangle**,*

*the **sum** of the squares of the two sides*

is equal

to the square of the hypotenuse



Pythagoras - Πυθαγόρας

Pythagorean triples **trójki pitagorejskie**

List of the First Few

Here is a list of the first few Pythagorean Triples (**not** including "scaled up" versions mentioned below):

(3,4,5)	(5.12,13)	(7.24,25)	(8.15,17)	(9.40,41)
(11.60,61)	(12.35,37)	(13.84,85)	(15.112.113)	(16.63,65)
(17.144.145)	(19.180.181)	(20.21,29)	(20.99,101)	(21.220.221)
(23.264.265)	(24.143.145)	(25.312.313)	(27.364.365)	(28.45,53)
(28.195.197)	(29.420.421)	(31.480.481)	(32.255.257)	(33.56,65)
(33.544.545)	(35.612.613)	(36.77,85)	(36.323.325)	(37.684.685)
... infinitely many more ...				

Example: scale **3,4,5** by 2 gives **6,8.10**

Which also fits the formula $a^2 + b^2 = c^2$:

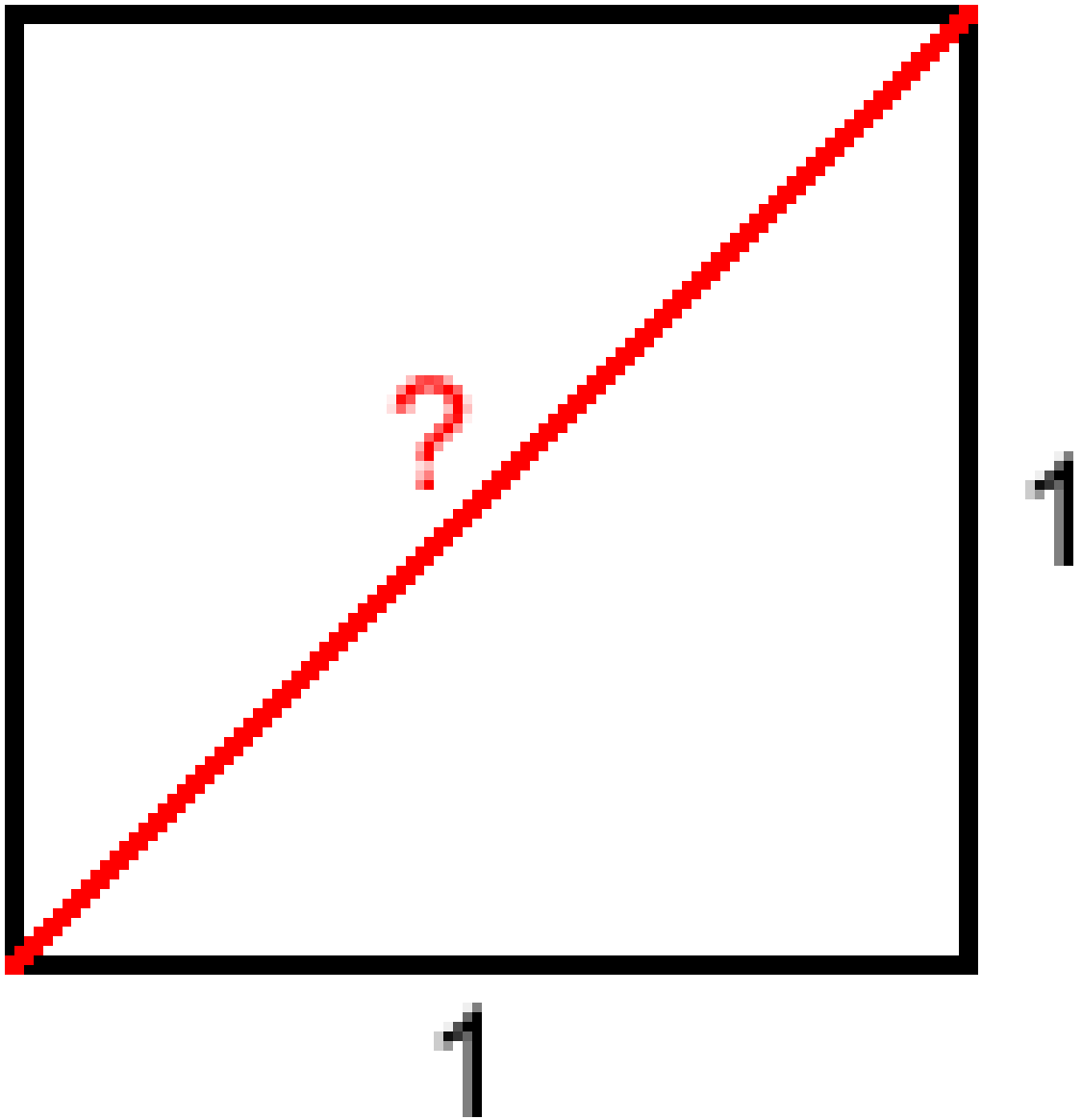
$$6^2 + 8^2 = 10^2$$

$$36 + 64 = 100$$

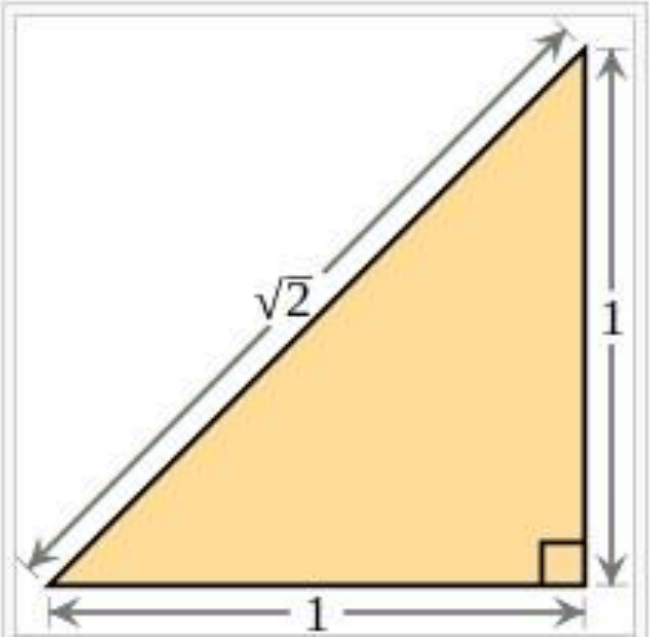
The Dangerous Ratio

Imagine a simple square shape, each side 1 unit in length.

How long is the square's diagonal?



The Dangerous Ratio



Square root of 2 is an **irrational number**

square root(2) =

1.41421356237

Pythagoras - Πυθαγόρας

Tired of finding no one who would listen to his learning, Pythagoras bought himself an audience. He found an urchin, a poor and ragged little fellow, and offered him a bribe. He would pay three oboli for every lesson the boy mastered.

To the urchin this was indeed a bargain. By sitting in the shade for a few hours, and listening attentively to this wise man, he could make better wages than in a whole day's work in the hot sun. Naturally, he concentrated hard while Pythagoras introduced him to mathematical disciplines.

From the simple calculations of the rope-stretchers, to the methods of the Phoenician navigators, to ahstract rules and reasoning, Pythagoras led his pupil on. Soon the subjects became so interesting that the boy begged for more and more lessons.

At this point, Pythagoras explained that he, too, was a poor man, and paying someone to listen was getting to be very expensive. So they reached another bargain. The boy had saved enough to pay Pythagoras for his future lessons. The story doesn't prove that Pythagoras began to collect a following this way. But it shows the fascination of the new game of string, straightedge, and shadows, and forecasts his great role as its teacher

Euclid & Eratosthenes

A famous geometer wrote down the whole ***theoretical*** subject in the best-selling mathematics text of all time,

and soon after,

another geometer performed the **most** spectacular ***practical feat***. *He used a* ***shadow*** to measure not just a pyramid, but the whole round earth!

These two events took place in the view Greek capital of the age-old land of Egypt

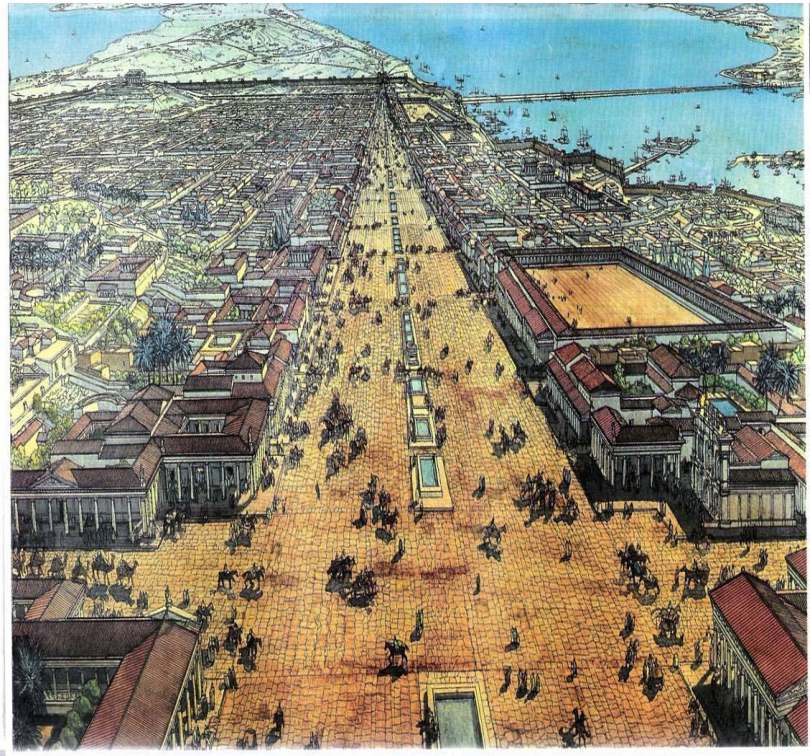
Alexandria

Founded by Alexander the Great and named for him, Alexandria had become the leading metropolis of the ancient world

Alexandria

Famous for it's **Light-house**
(one of the seven wonders)

and it's **Library**



Euclid – Ευκλείδης

This cosmopolitan city was the gathering place for the best scholars and scientists of the age. Savants from many lands made their discoveries in the "museum"- a graduate school that carried on studies in literature, medicine, astronomy, and mathematics.

The accumulated learning of the past was stored in the great annexed **Library**, with nearly a million books on scrolls.

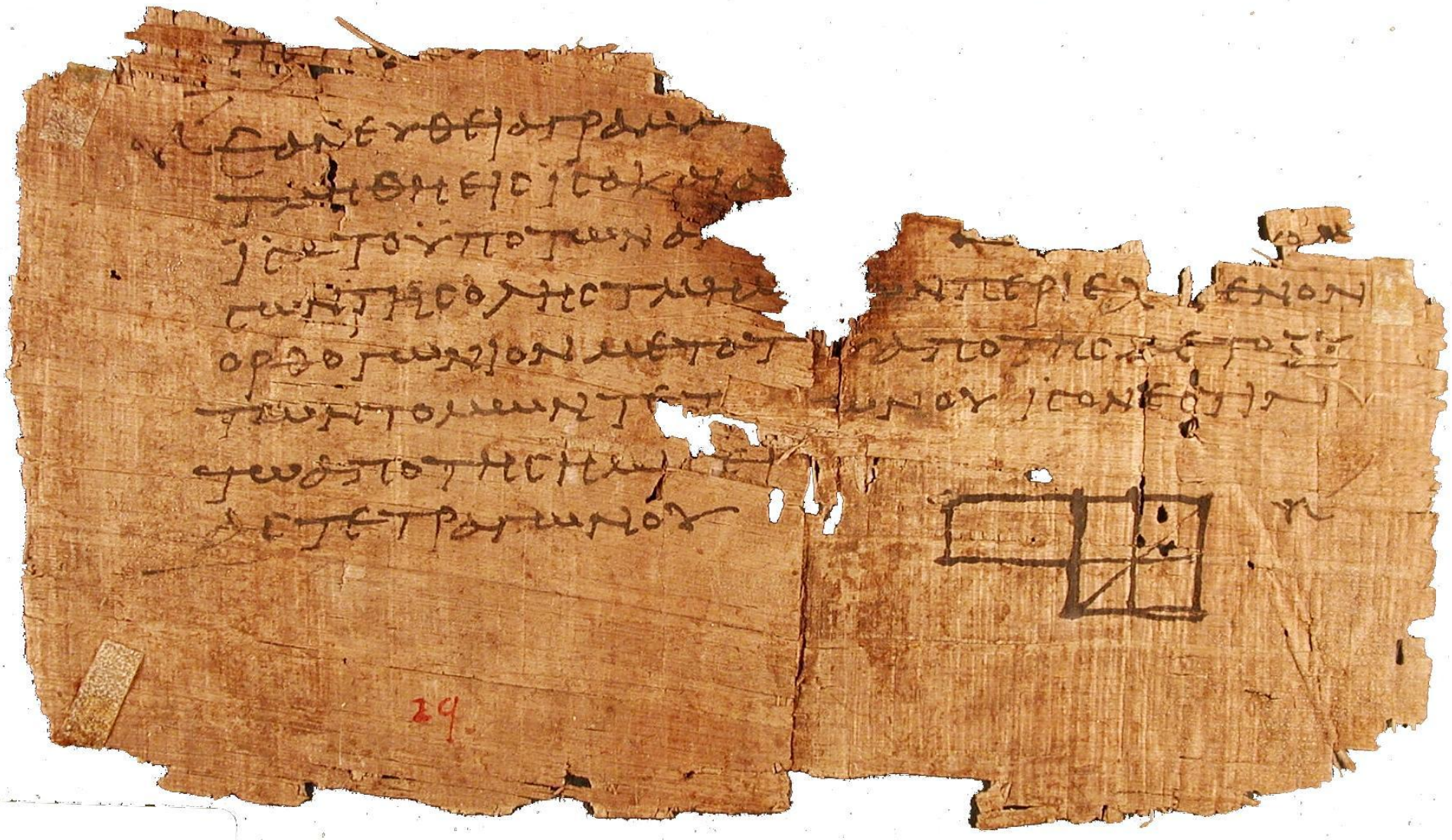
One of the first scholars and most famous was

Euclid!

His work **Elements** (13 books) are the most published worldwide after Bible. Even today, in almost all countries, are taught in schools

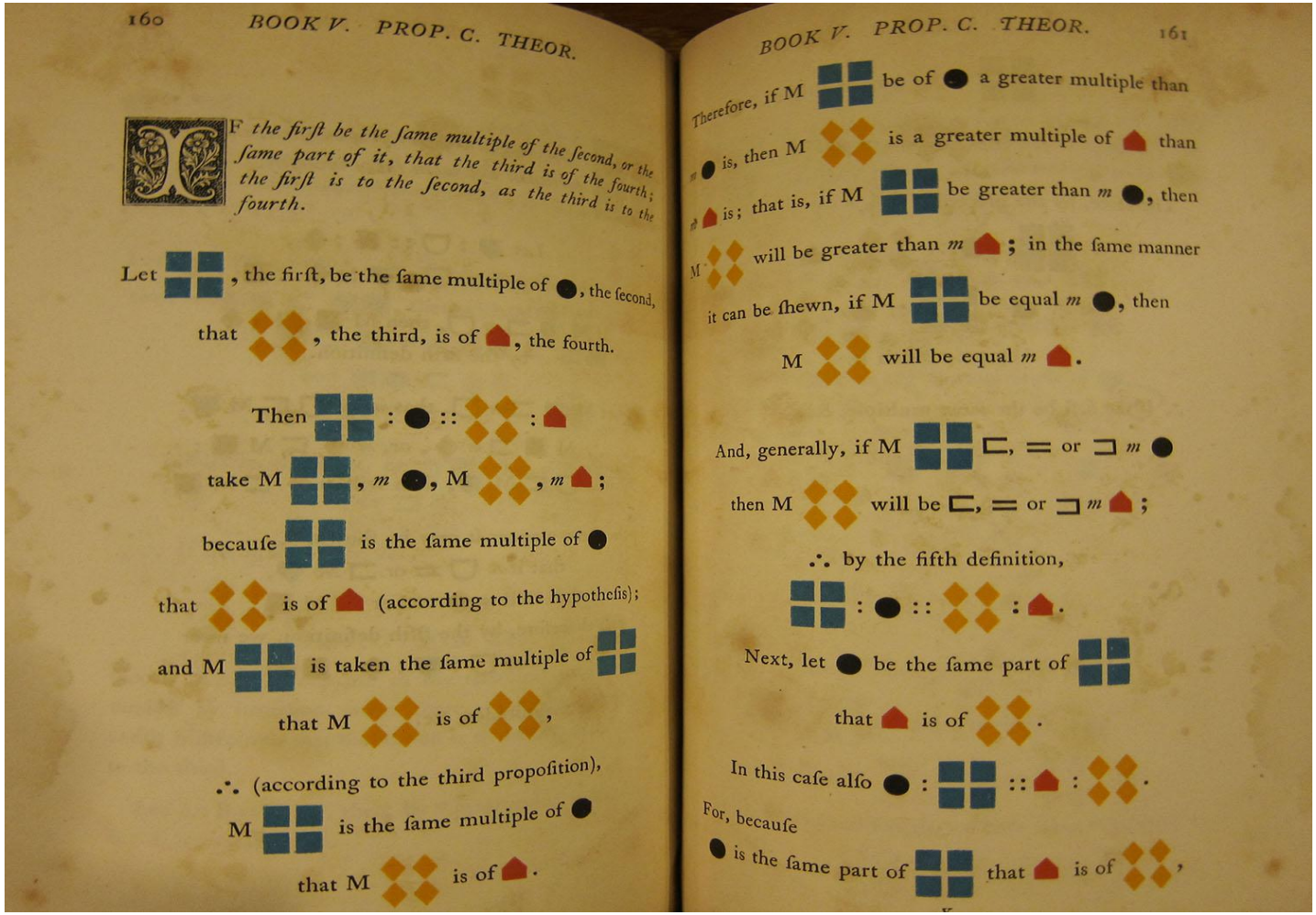
Euclid – Ευκλείδης

These 13 books tell the story of mathematics



Euclid – Elements

Euclid's elements in an old school book



160 BOOK V. PROP. C. THEOR.

IF *the firſt be the ſame multiple of the ſecond, or the ſame part of it, that the third is of the fourth; the firſt is to the ſecond, as the third is to the fourth.*

Let , the firſt, be the ſame multiple of , the ſecond,

that , the third, is of , the fourth.

Then : :: :

take M , *m* , M , *m* ;

becauſe is the ſame multiple of

that is of (according to the hypotheſis);

and M is taken the ſame multiple of

that M is of ,

∴ (according to the third propoſition),

M is the ſame multiple of

that M is of .

BOOK V. PROP. C. THEOR. 161

Therefore, if M be of a greater multiple than

is, then M is a greater multiple of than

is; that is, if M be greater than *m* , then

M will be greater than *m* ; in the ſame manner

it can be ſhewn, if M be equal *m* , then

M will be equal *m* .

And, generally, if M ⊏, = or ⊐ *m*

then M will be ⊏, = or ⊐ *m* ;

∴ by the fifth definition,

: :: : .

Next, let be the ſame part of

that is of .

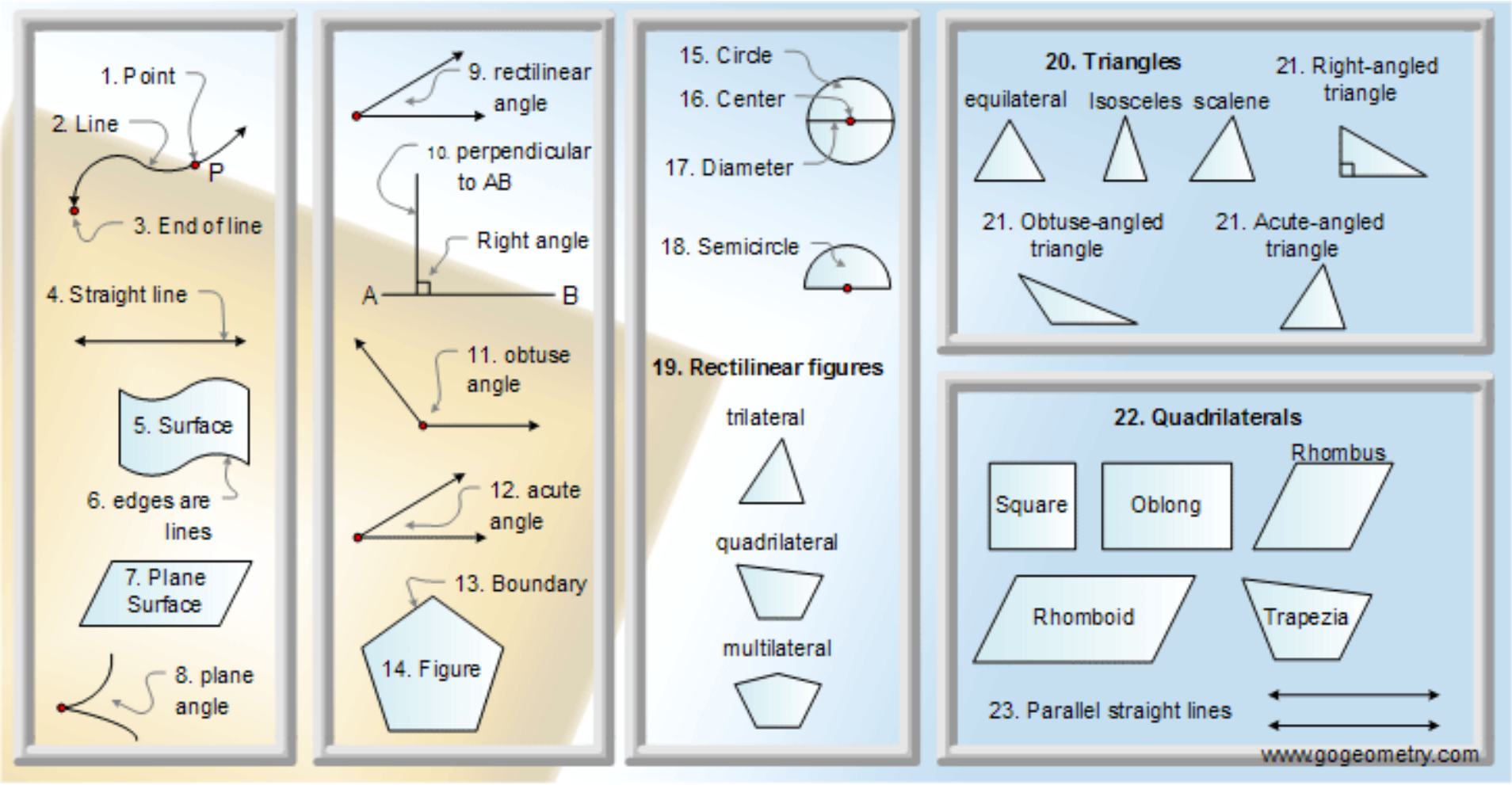
In this caſe alſo : :: : .

For, becauſe

is the ſame part of that is of ,

Euclid – Elements

Euclid elements, Book I 23 definitions



Euclid – Elements

γεωμετρίᾳ δόξαν αὐτοῦ λαβόντος. ἐπὶ δὲ τούτοις 15
Πυθαγόρας τὴν περὶ αὐτὴν φιλοσοφίαν εἰς σχῆμα
παιδείας ἐλευθέρου μετέστησεν, ἄνωθεν τὰς ἀρχὰς
αὐτῆς ἐπισκοπούμενος καὶ ἀΰλως καὶ νοερῶς τὰ θεω-
ρήματα διερευνώμενος, ὃς δὴ καὶ τὴν τῶν ἀλόγων
πραγματείαν καὶ τὴν τῶν κοσμικῶν σχημάτων σύστα- 20
σιν ἀνεῦρεν. μετὰ δὲ τοῦτον Ἀναξαγόρας ὁ Κλαζο-

Eratosthenes - Ερατοσθένης

The first librarian of the famous Library of Alexandria was a Creek named **Eratosthenes**.
A universal mind, he was a mathematician, a specialist in history, an astronomer, a geographer, and a poet besides.

At around **250 B.C.** he did something almost incredible in those times.

Eratosthenes **measured accurately** the girth of the planet he lived on!

Eratosthenes - Ερατοσθένης

Strange as it seems, he had a practical purpose in mind. As a great geographer, he understood that the earth was round, and he was mapping the known parts of it.
On his map of the world, Eratosthenes put all the data and distances he could get. The project was typical of that era, when the Mediterranean was becoming "one world" for the first time. It was one world of scientists; astronomers, mathematicians, and geographers in many lands were pooling their knowledge. And it was one world of trade, of ships and sailors, who needed maps.

To make his map more accurate and useful, Eratosthenes wanted to determine the width of a degree of latitude. But for that, he had to know the circumference of the earth.
How was he to measure it?

The inspiration came to him one day as he was traveling up the Nile

on a summer study trip to Syene (today Aswan)

Eratosthenes - Ερατοσθένης

He noticed with excitement that on the ***longest day*** of the year the noonday sun shone straight down a well at Syene, a town about 5000 stadia up the river from Alexandria.

He could see the shape of the sun reflected on the surface of the water at the bottom of the well. But from there northward to Alexandria where he lived, the sun never got directly overhead;

So Syene was really on the Tropic of Cancer! To a geographer that was most important, and he explored the region in order to draw the Tropic on his map

Eratosthenes - Ερατοσθένης

But the sight of the sun in the well fired his imagination even more. Just that single observation, plus his knowledge of geometry and his own active brain, told him how to determine the distance around the earth.
He did it by **means of a shadow** and some remarkably deductions.

Eratosthenes simply took the known distance between **Syene** and **Alexandria**, due north and then measured a single angle at the right place and the right time.
He made his historic measurement at Alexandria, at noontime on the longest day of the year. At that moment, he knew, the sun was shining straight down the well at Syene, **5000** stadia away, casting no shadow. But at Alexandria, where he stood, an upright post was casting a shadow

We can't say exactly how good his estimate was, because the 'stade' unit of length meant different things to different people.

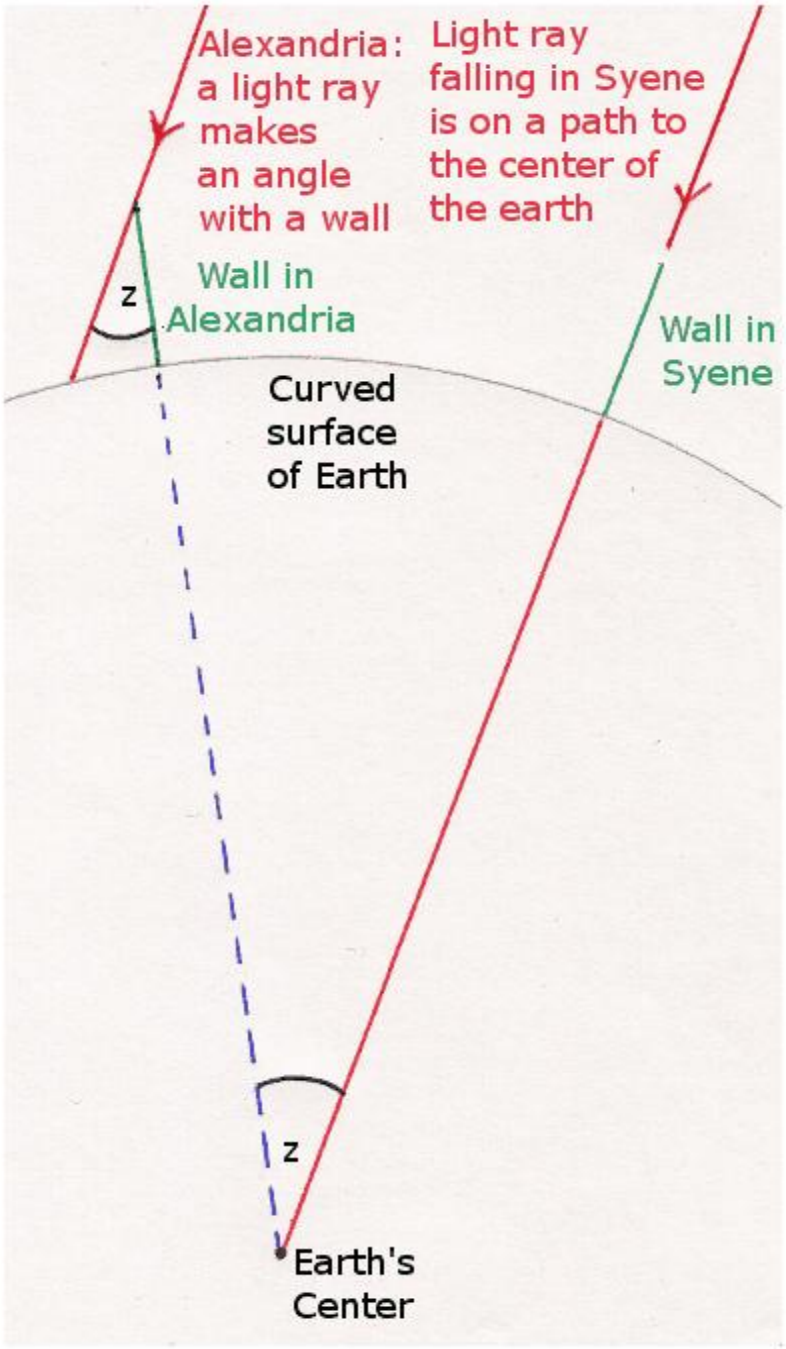
Whichever stade was used, Eratosthenes *overestimated* the size of Earth. Depending on which stade he used, we can say that his estimate was, at best, within 1% and, at worst, within about 30% of the value we use now.

(Our current value for Earth's polar circumference is 40,075.16 km or 24,901.55 miles)

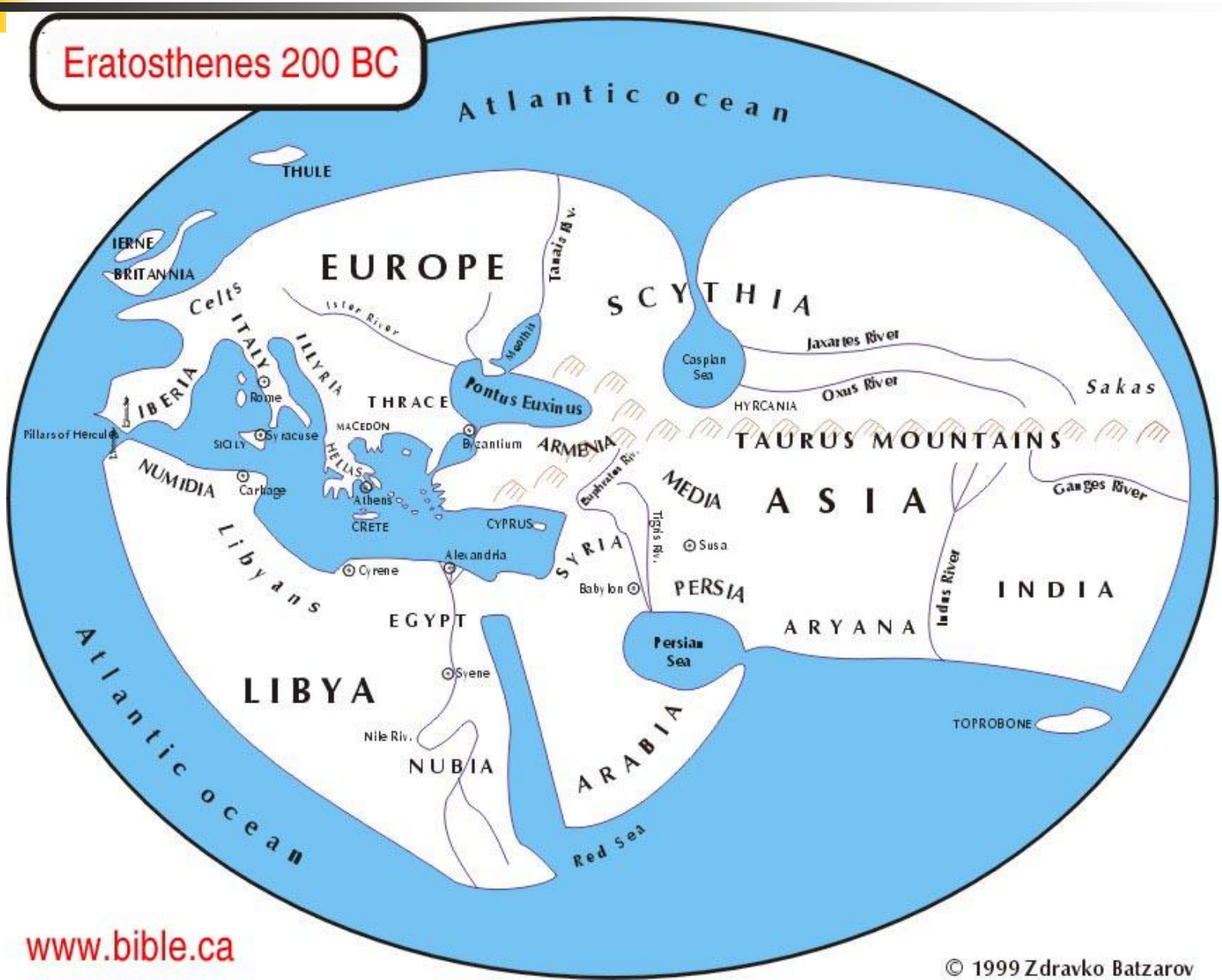
Whichever way you look at it,

this was an **enormous advance**

in an era when most people in the world had no idea that our planet is approximately spherical.



Map of Eratosthenes



Prime numbers Eratosthenes' Sieve

The Eratosthenes' sieve from 1 to 150:

~~1~~, [2], [3], ~~4~~, [5], ~~6~~, [7], ~~8~~, ~~9~~, ~~10~~,
[11], ~~12~~, [13], ~~14~~, ~~15~~, ~~16~~, [17], ~~18~~, [19], ~~20~~,
~~21~~, ~~22~~, [23], ~~24~~, ~~25~~, ~~26~~, ~~27~~, ~~28~~, [29], ~~30~~,
[31], ~~32~~, ~~33~~, ~~34~~, ~~35~~, ~~36~~, [37], ~~38~~, ~~39~~, ~~40~~,
[41], ~~42~~, [43], ~~44~~, ~~45~~, ~~46~~, [47], ~~48~~, ~~49~~, ~~50~~,
~~51~~, ~~52~~, [53], ~~54~~, ~~55~~, ~~56~~, ~~57~~, ~~58~~, [59], ~~60~~,
[61], ~~62~~, ~~63~~, ~~64~~, ~~65~~, ~~66~~, [67], ~~68~~, ~~69~~, ~~70~~,
[71], ~~72~~, [73], ~~74~~, ~~75~~, ~~76~~, ~~77~~, ~~78~~, [79], ~~80~~,
~~81~~, ~~82~~, [83], ~~84~~, ~~85~~, ~~86~~, ~~87~~, ~~88~~, [89], ~~90~~,
~~91~~, ~~92~~, ~~93~~, ~~94~~, ~~95~~, ~~96~~, [97], ~~98~~, ~~99~~, ~~100~~,
[101], ~~102~~, [103], ~~104~~, ~~105~~, ~~106~~, [107], ~~108~~, [109], ~~110~~,
~~111~~, ~~112~~, [113], ~~114~~, ~~115~~, ~~116~~, ~~117~~, ~~118~~, ~~119~~, ~~120~~,
~~121~~, ~~122~~, ~~123~~, ~~124~~, ~~125~~, ~~126~~, [127], ~~128~~, ~~129~~, ~~130~~,
[131], ~~132~~, ~~133~~, ~~134~~, ~~135~~, ~~136~~, [137], ~~138~~, [139], ~~140~~,
~~141~~, ~~142~~, ~~143~~, ~~144~~, ~~145~~, ~~146~~, ~~147~~, ~~148~~, [149], and ~~150~~.

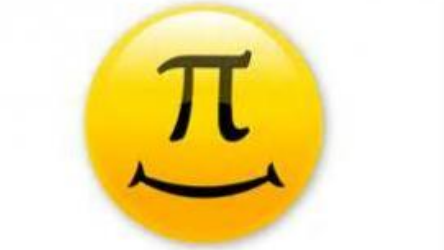
A simple formula

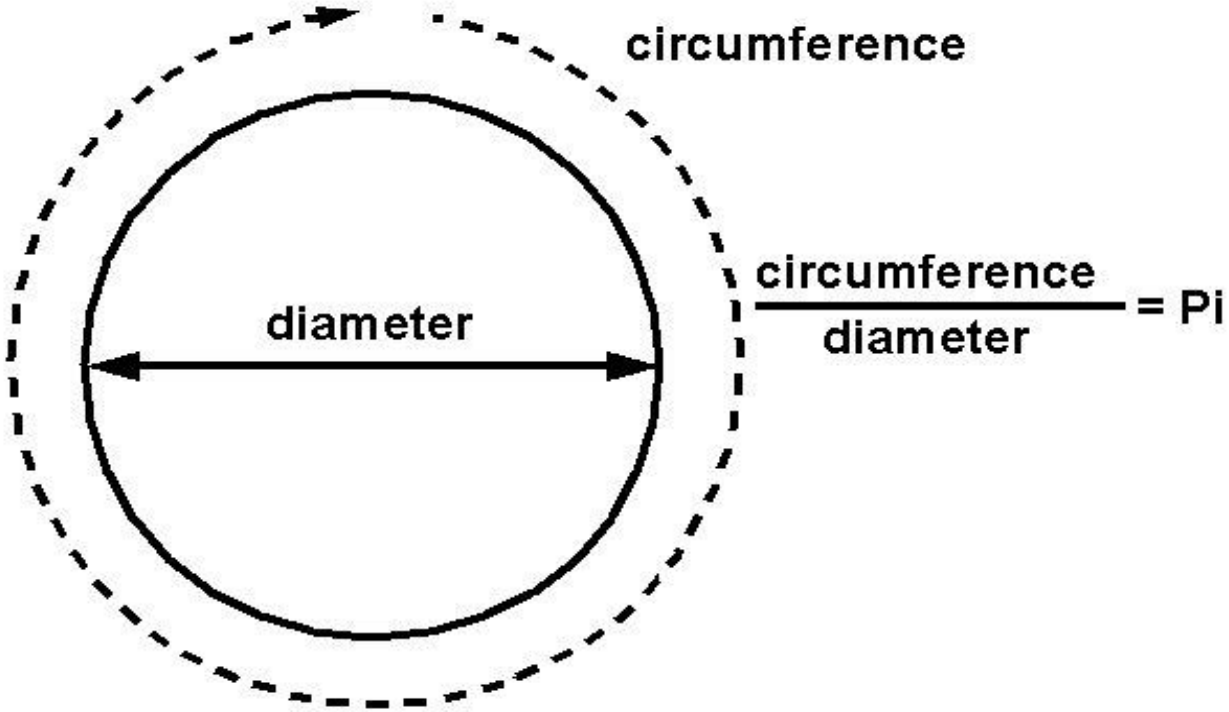
$$1 + 2 + 3 + \ldots + v = \frac{v \times (v+1)}{2}$$

Proving that 1+2+3+...+n is n(n+1)/2

The sum of the first 10 numbers is:
1+2+3+4+5+6+7+8+9+10=
10(10+1)/2=
110/2=55

π (pi)





π

3.14159
26535897932384626433832795
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φ (phi)

Mathematics of Phi, 1.618, the Golden Number

May 16, 2012 by Gary Meisner — 27 Comments

Phi, Φ, 1.618…, has two properties that make it unique among all numbers.

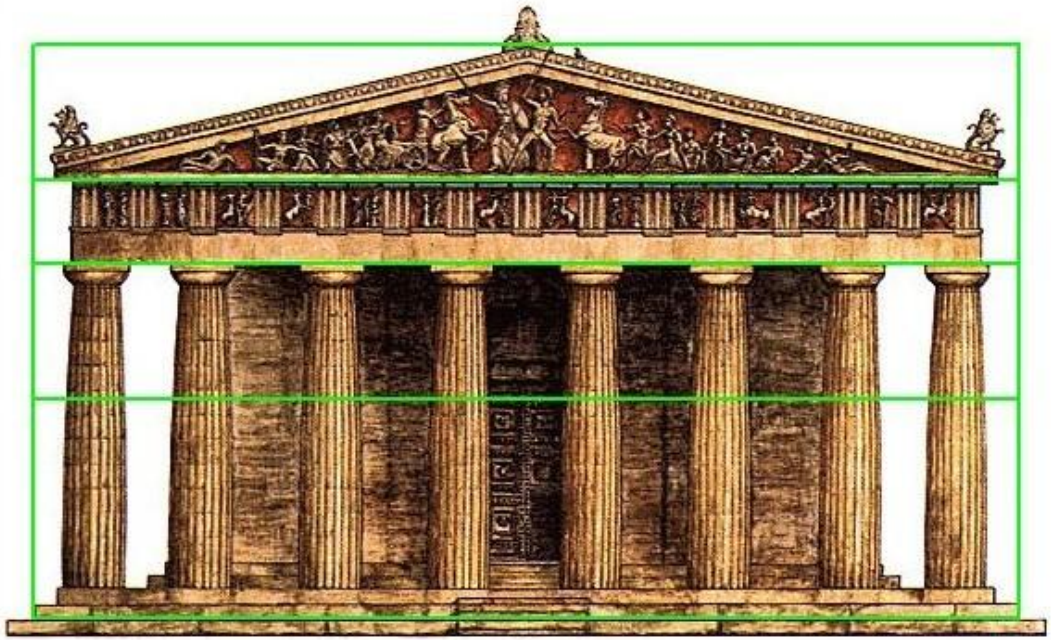
- If you square Phi, you get a number exactly 1 greater than itself: 2.618…, or

$$\Phi^2 = \Phi + 1.$$

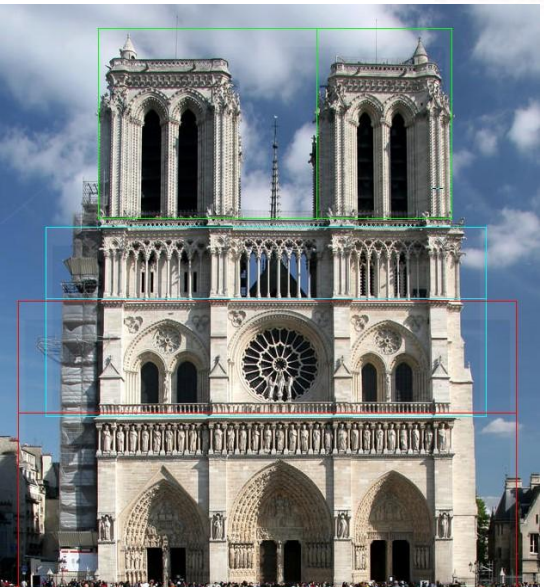
- If you divide Phi into 1 to get its reciprocal, you get a number exactly 1 less than itself: 0.618…, or

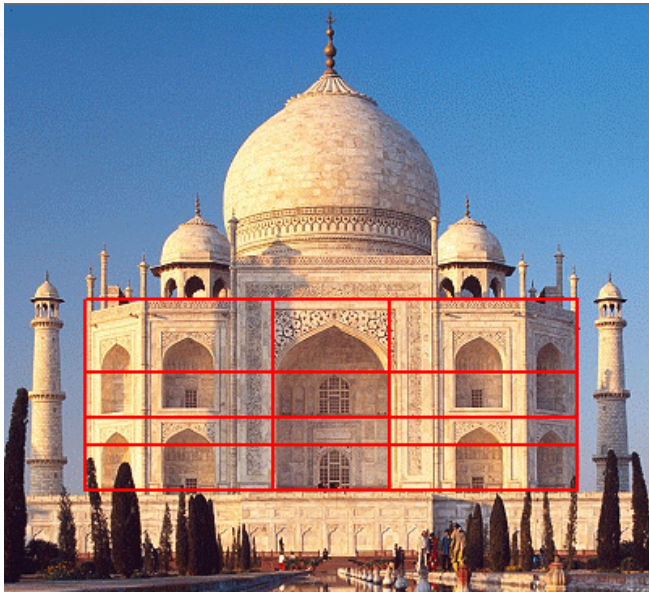
$$1 / \Phi = \Phi - 1.$$

φ (phi)

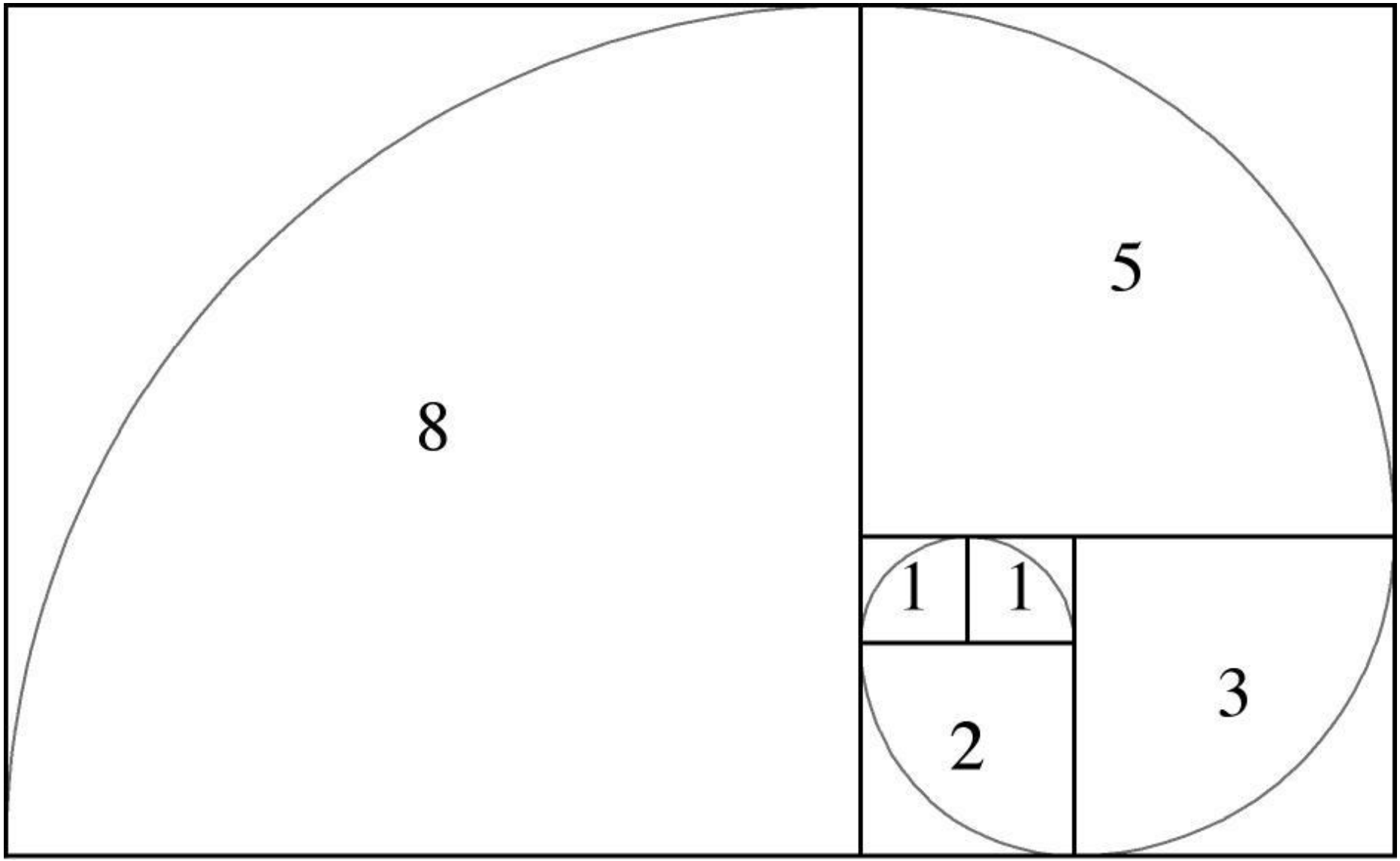


THE PARTHENON





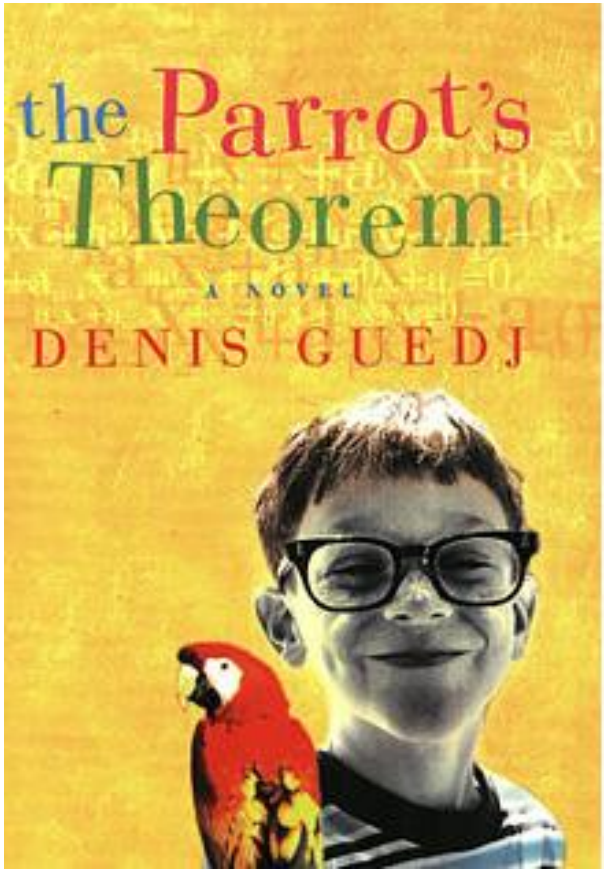
Fibonacci Sequence



The Parrot's Theorem

The Parrot's Theorem

is a French novel written by **Denis Guedj** and published in 1998.



The Parrot's Theorem

Plot summary

The plot revolves around a household in Paris:

Mr Ruche, an elderly wheelchair-using bookseller; his employee and housemate Perrette; and Perrette's three children - teenage twins and young Max who is deaf. Max liberates a talking parrot at the market and Mr Ruche receives a consignment of mathematical books from an old friend, who has lived in Brazil for decades without any contact between the two.

The household sets up its own exploration of mathematics in order to crack the code of the last messages from Mr Ruche's old friend, now apparently murdered.

Mathematical topics covered in the book include **primes** and **factors**; **irrational** and **amicable** numbers; the **discoveries of Pythagoras**, **Archimedes** and **Euclid**; and the **problems** of *squaring the circle* and *doubling the cube*.

The mathematics is **real mathematics**, woven into an historical sequence as a series of intriguing problems, bringing their own stories with them.

Magic Maths

Choose any two digit number, add together both digits and then subtract the total from your original number.*

When you have the final number look it up on the chart and find the relevant symbol. Concentrate on the symbol

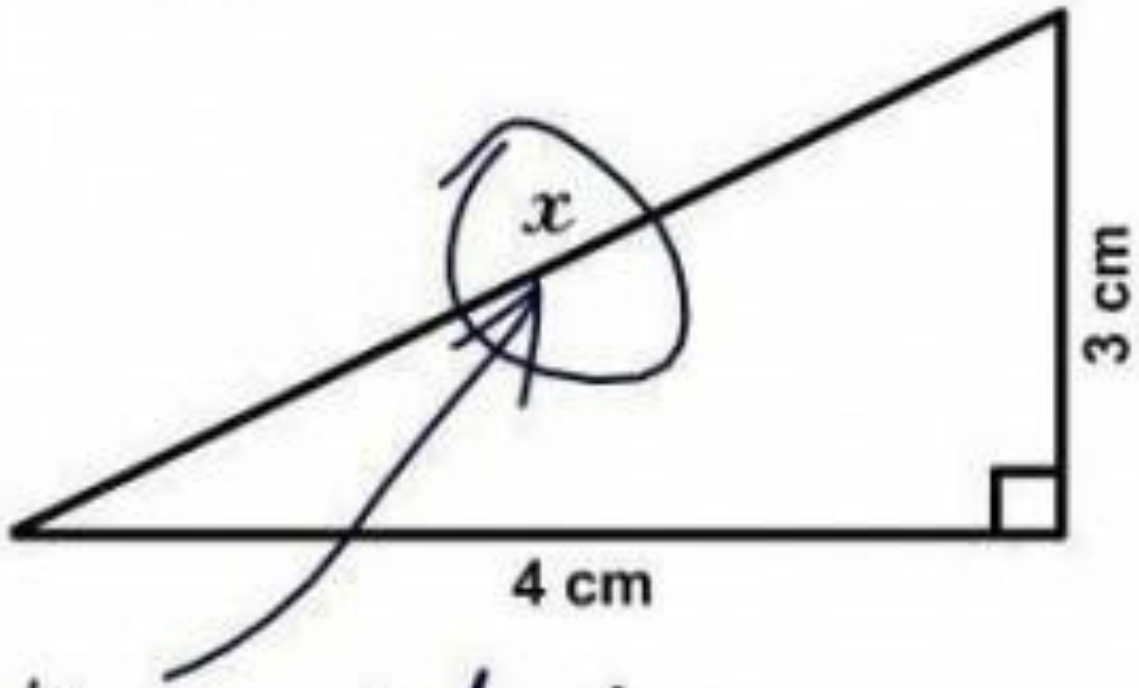
* For example if you chose 23: 2+3 = 5. 23 minus 5 will give you your answer.


99 79 59 39 19
98 78 58 38 18
97 77 57 37 17
96 76 56 36 16
95 75 55 35 15
94 74 54 34 14
93 73 53 33 13
92 72 52 32 12
91 71 51 31 11
90 70 50 30 10
89 69 49 29 9
88 68 48 28 8
87 67 47 27 7
86 66 46 26 6
85 65 45 25 5
84 64 44 24 4
83 63 43 23 3
82 62 42 22 2
81 61 41 21 1
80 60 40 20 0

Magic Maths

A very clever answer!!

3. Find x.



Here it is

105th Primary School of Thessaloniki

presented by Giannis Michalis